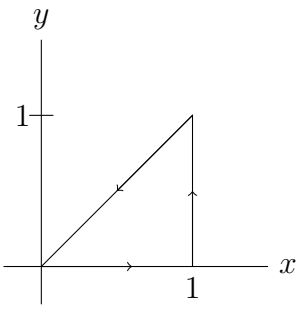
Multivariable Calculus

Quiz 13 **SOLUTIONS**

1) Compute $\oint_C (x+y)dx - (x^2+y^2)dy$ over the triangle shown below.



Solution: By Green's Theorem,

$$\begin{aligned}
\oint_C (x+y)dx - (x^2+y^2)dy &= \iint_D (-2x-1)\ dA \\
&= -\int_0^1 \int_0^x (2x+1)\ dy\ dx \\
&= -\int_0^1 (2xy+y)\Big|_{y=0}^{y=x}\ dx \\
&= -\int_0^1 (2x^2+x)\ dx \\
&= -\left(\frac{2}{3}x^3 + \frac{1}{2}x^2\right)\Big|_0^1 \\
&= -\frac{7}{6}.
\end{aligned}$$

TURN OVER

2) Compute $\oint_{\mathcal{C}} \langle 2x + y^2,\ 2xy + 3y \rangle \cdot d\vec{r}$ over *any curve* $\mathcal{C}$ for which Green's Theorem applies.

Solution: Let $\mathcal{D}$ be the region enclosed by $\mathcal{C}$. Then by Green's Theorem,

$$\begin{aligned} \oint_{\mathcal{C}} \langle 2x + y^2,\ 2xy + 3y \rangle \cdot d\vec{r} &= \iint_{\mathcal{D}} (2y - 2y)\ dA \\ &= \iint_{\mathcal{D}} 0\ dA = 0 \end{aligned}$$